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A Study of Population Growth¹

Introduction

System Dynamics provides us with a very powerful tool for studying the behavior of complex systems over time. Once a model has been constructed, simulations can be run and model values can be recorded on graphs and tables. The graphs are usually plotted with time on the horizontal axis, and are therefore known as *behavior-over-time graphs* (BOTGs).

Reference Behavior

You may be asked to sketch “reference” behavior graphs before developing a model and running the simulations. A reference graph is usually a BOTG. You should sketch the reference graph on paper, describing the behavior you are trying to model. This can often be quite difficult when you don’t understand the problem very well, but it is crucial that you make some attempt to describe the simplest possible behavior, and then develop a very simple model to simulate this behavior. Even if your guess is wrong it will focus your attention on the important behavior of the system, and make you think much more critically about it.

Feedback Loops

Another critical component of System Dynamics is the consideration and study of “feedback loops” within the complex system. We are typically taught to think about “cause and effect,” but in reality, the “effect” often loops back to influence its own cause. These relationships are very hard to estimate without a computer simulation, especially when they occur over long periods of time or over distances in space. Examples might include changes in interest rates by the federal government as a result of excessive spending by the general population or as a result of inflation. These situations may be “caused” by the behavior of the general population, but the “effect” (higher interest rates) may not happen until months later. The decision will be made in Washington, D.C. and reported in the paper, but most people will never realize that their actions led to this result. To make matters more complicated, people may begin to buy more because of *rumors* that interest rates will be going up, so they better buy now before prices increase. Their actions therefore act to aggravate the situation and eventually “cause” interest rates to go up!

¹ This lesson was written by Diana Fisher and Ed Gallaher.

The First Population Model

This tutorial will concentrate on a study of population because it is a model whose elements are relatively easy to understand. We start with an overview of the important factors in this model. In a population model the focus of study is to observe changes in the number of people in the group at any point in time. Consequently, we will have a special icon in our model identified as population.

We all know there are many factors that affect the change in the number of people in a population. In order to keep this model simple we will limit our study to only two flows – one for *births* and one for *deaths*.

Using Your Journal



Draw three different possible population graphs (no numbers) that span 100 years. Label the horizontal axis *years* and the vertical axis *population*. Give three, one sentence explanations (one for each graph) of what might cause each type of pattern.

Part I: The Birth Flow and Reinforcing Feedback Loop

Consider the births. Since births only increase a population we will design part of our model to include this fact. To keep the model very simple, let us assume that each year the population will be increased by a certain number of babies and that the birth rate (or birth fraction) value will not change over the entire simulation time for this model. As a convenient (and reasonable for the US) value, let's assume that there are 15 births for every 1000 people in the population each year. We will identify that fraction as the birth fraction.

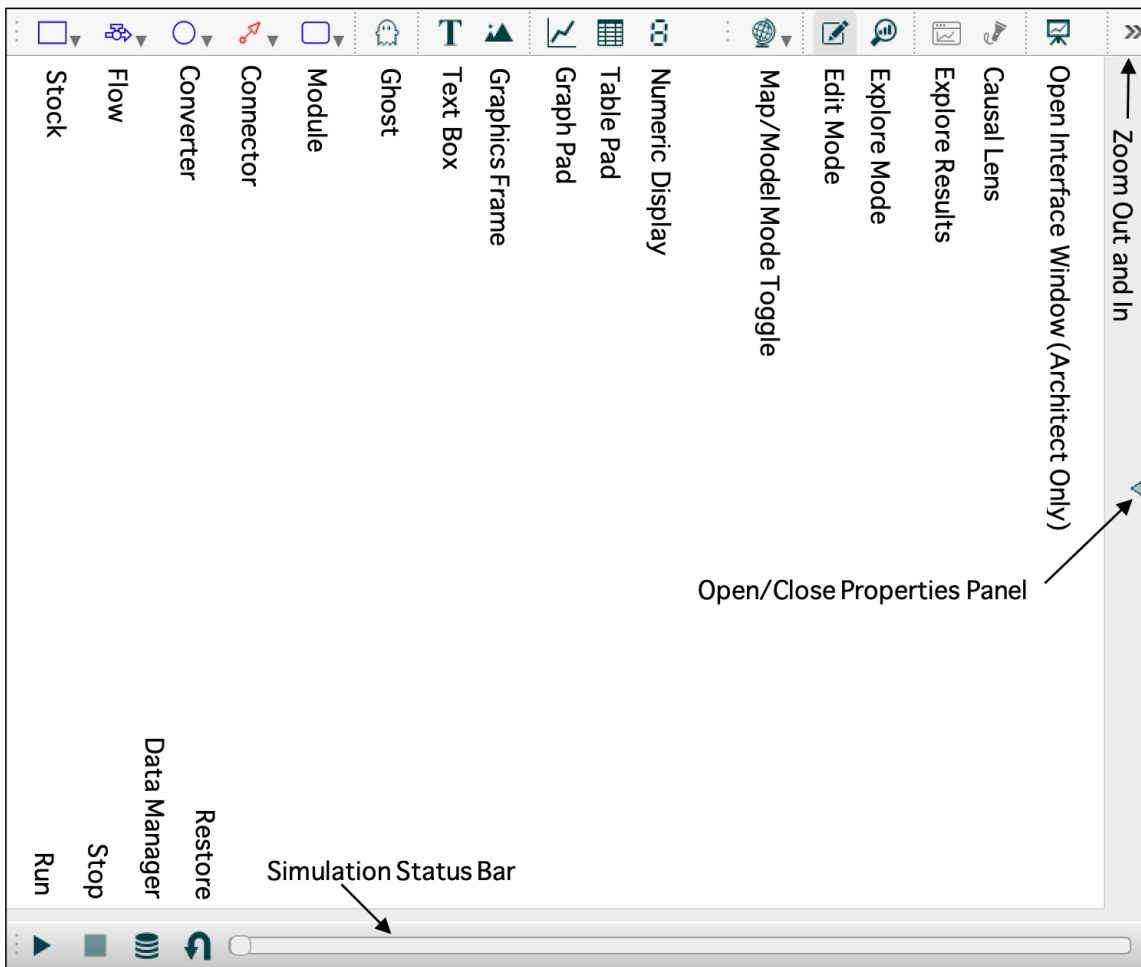
Births add people to a population, which, because the population is now increased, causes even more births to occur. A loop connection is created between the number of births and the population. This loop would be identified as a "reinforcing" feedback loop since it will lead to an ever-increasing number of people in the population.

Let's start the population at 100,000 persons, as we begin our study. We would like to run the simulation for 50 years, and see what is happening every year.

Note: It is extremely important, when creating models, to make sure the units for each component are consistent. In order to establish the proper habit, unit specification will be REQUIRED when defining each model component value or formula.

Constructing the Model

1. Open Stella by double-clicking on the Stella icon.
2. Stella has several layers. For this tutorial we need to be in the Modeling layer. When you open the application you will automatically be in the Map layer. Click on the Globe icon and change it to χ^2 in the menu bar for the Stella window. (See the sample window in item 3, below.)
3. You should now have a blank area, a window, labeled "Stella Architect" (or Stella Professional). Note each of the icons and the names for each icon. A sample window is shown here.





4. Select the Stock icon (first icon from the left in the toolbar) by clicking on it. Release the click.
5. Slide the arrow onto the window *without* holding down the button.
6. Position the Stock in the center of the screen, about 1 inch below the top border of the window. Then click once to deposit it.

7. With the Stock selected, type *Population*. If you make a mistake, backspace to correct it.
8. Select the Flow icon (located to the right of the Stock icon on the toolbar) by clicking once on it. Be sure to release your click after you select the icon. Move the cursor onto the screen. Position it about two inches to the left of *Population*, on a horizontal line that slices the Stock in half.
9. Click and hold the mouse button. Then, drag the cursor to the right until it makes contact with the left wall of the Stock (the Stock should turn blue on contact). Release the click.
10. While the Flow is selected (dark), name it *Births*.
11. Select the Converter icon (third icon from the left on the toolbar). Deposit it about an inch or so below and about 2 inches left of *Population*. Name it: *Birth Fraction*.
12. Select the Connector icon (it looks like an arrow with a circle at the tail). Place the cursor *inside* the converter for *Birth Fraction*. Click and hold. Drag the cursor out of the Converter until it makes contact with the circular part of the *Births* flow (the latter will turn blue). Release your click.
13. Select the Connector icon again. Place the connector cursor inside the *Population* stock. Click and hold. Drag the cursor out of the Stock and make contact with the circular part of the *Births* flow (the latter will turn blue). Release your click.
14. If you haven't done so already select Save from the File menu. Give your model a name and then click on Save.
15. (Make sure you are in the Model – not Map - view.) Use the cursor (arrow) to double click on the circular part of the *Births* flow. A new panel should appear. This panel will allow you to define how *Births* are to be calculated during the simulation.
16. Click once on *Population* under "Required Inputs." Then locate and click on the * operator key. Then click on *Birth Fraction*. You should have the equation:

$$Births = Population * Birth Fraction$$


Births is a flow. The software does not allow the modeler to designate flow units. Once the stock units are specified (i.e., as {people} - see item 18 below) and the simulation time units are specified (i.e., as {years} - see item 21 below) the software will automatically generate the correct units for the flow {people/year} (with no curly brackets).

Click the  button in the bottom-right corner of the panel when done. Click the  button in the middle of the left side of the panel to close the panel, if desired.

Question: What does the equation mean (in your own words)? Do not just repeat the equation. Explain what it means in the population we are studying. I'll get you started: The equation means that we can figure out how many children are born each year by determining...

What assumption is being made here about the growth of the population over time? In other words, is the birth fraction changing over time? Why or why not? (*Hint:* Look back on the first two pages of this tutorial for the word "assume.")

17. With the cursor (arrow), double click on *Population*.

18. Type in the number $1e5$ or 100000 {people}². Click  when you're finished.

Question: What does this number represent?

19. With the Arrow, double click on *Birth Fraction*. Define it to be $15/1000$ {people/people/year} or just {1/year}. Click the green check when done.

Question: What does the value "15/1000" represent (in your own words)?

Setting up the Simulation and Graph

20. While in Edit mode double click in the modeling window. A Model Settings panel will appear.

21. Keep the "Start Time" as 0. Set the "Stop Time" to 50. Change the DT to 1.00 or 1/1 depending upon whether fractional DT has a check mark. Specify "Years" for "Time Units." Close the panel by clicking in the model window.

² Note: I will indicate units in curly brackets in these lessons (for convenience), but you should always place the units in the correct location of the definition panel (without the curly brackets).

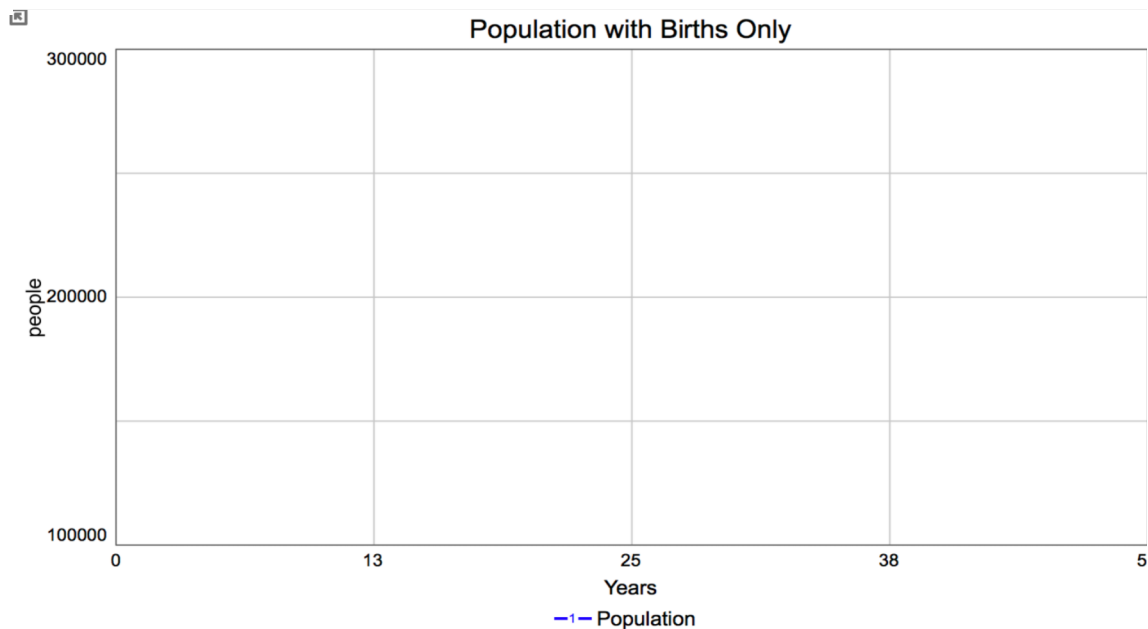
Question: What is the purpose of changing these numbers? What do these numbers mean? (Explain the 0, 50, and the 1.)

22. Click on the Graph icon in the toolbar. Place the icon two inches below and to the right of the *Population* stock. Click once to deposit the graph icon and it will automatically open. Once the graph opens double click on it.

23. Click on the green + button below the Series List box. A window will open that will allow you to select the variable you want to place in the graph. Click on *Population*. (Alternatively, you could just type the variable name in the box next to “Variable” below the Series list box, and click the + button to add it to the Series List box.

24. Select Save from the file menu to save all the work you've done so far.

25. Sketch a graph of the reference behavior for the population values on the blank grid below. The reference behavior is the behavior that you think will happen to the population over time. (Make the sketch with a dotted curve, rather than a solid curve.)³



Run the simulation by selecting the Model menu sliding down to Run in that menu. Sketch the graph (using a solid curve) that Stella produced on the same grid (above) in which you drew your prediction.

³ Note: If your graph pad does not show a 4 x 4 grid, scroll to the bottom of the graph definition panel. Click “Grid Options.” Enter 5 for “X Grid Lines” and 5 for “Y Grid Lines.”

How did your prediction compare to the graph produced by the computer?

Part II: The Death Flow and Balancing Feedback Loop

It does not make sense that a population would only increase. We know that deaths occur each year, causing a total population value to remain more stable. As our first experiment, and to keep the model very simple, let us assume that deaths occur from old age only. Let's also assume that the average life span of the population is 76 years (for both men and women). We will add a component to our population that is labeled "Lifespan." We will assume that the value we will assign will remain constant over the entire simulation. This is a major assumption, but not an unreasonable place to start in the study of our first population. The number of people who die each year will be determined by calculating $1/76$ of our total population. In other words, the people in the population reside in the stock for an average of 76 years. So $1/76$ of the total population will flow out (as deaths) each year of the simulation.

We know that deaths have the effect of reducing the number of people in a population (deaths are determined by taking away a percentage of the total population). If, for some reason, the population (stock value) became larger, the number of deaths (outflow) would become larger. An increase in the number of deaths would drive the population (stock value) downward. The effect of this cycle is to bring the population into balance. This is referred to as "balancing" feedback.

Constructing More of the Model

26. (Remember to change back to "Edit Mode" after running the simulation, when you want to modify the model.) Select the Flow icon again. Position the flow cursor *inside* the *Population* Stock. Click and hold. Then, drag the icon out of the Stock (through the right wall) to a distance about two inches from the wall. Release the click.

27. Name the outflow *Deaths*. Click anywhere on the blank screen to deselect *Deaths*.

28. Select the Converter icon again. Deposit a converter below, and three inches to the right of *Population*. Name it: *Lifespan*.

29. Select the Connector icon. Use it to link *Population* to *Deaths*.

30. By depressing the "option" (Mac) or "alt" (Win) key you can recall the previously selected tool. Do that to recall the Connector icon. Place the connector cursor *inside* the Converter for *Lifespan*. Click and hold. Use it to link *Lifespan* to *Deaths*.

31. With the cursor, double click on *Deaths*. Define *Deaths* as the product of *Population* and $1/Lifespan$.

Question: What does the *Deaths* equation look like? (Write it below.)

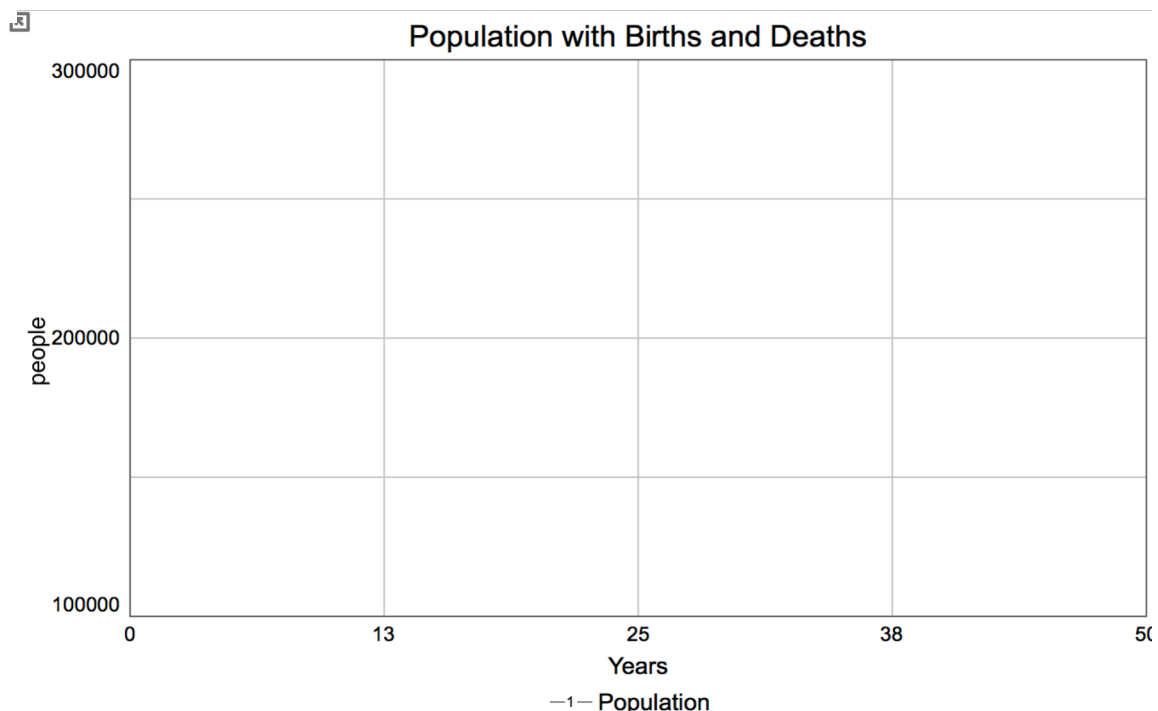
Check to make sure Stella has specified the expected flow units. Click when done.

32. With the cursor, double-click on *Lifespan*. Define it to be 76 {years}. Click .

Question: What does the value "76" represent (in your own words)?

33. Now would be a good time to Save your model again.

34. Sketch a graph of the reference behavior for *Population* on the blank grid below. (Make the sketch with a dotted curve, rather than a solid curve).



Run the simulation by clicking on the triangle in the lower left of the Stella window. Sketch the graph (using a solid curve) that Stella produced on the same grid (above) in which you drew your prediction. (Be careful. Stella may have changed the vertical scale for its graph. You will learn how to control this a little later.)

Describe the difference between the model output and your reference behavior pattern.

Creating a Table for Numerical Output

35. Click once on the Table icon in the toolbar. (It is located to the right of the graph icon.) Deposit the table icon below the "Graph" icon by clicking once. Once the table opens double click on it.

36. Enter *Population* into the table by following the same procedure as described in step 23 used to add the *Population* variable to the graph pad.


37. Run the simulation. Record the population value at 50 years (here and on another piece of paper to use for the next assignment).⁴

(*Note:* You may resize the width of each column in the table by double clicking on the table to open the table Properties panel. Select the paint roller button at the bottom of the panel. In the "Options" section of the panel uncheck "Auto-size column width" and enter a specific value for the desired column width.)

38. Save your work to your disk. (*Note:* All diagrams, graphs, and tables are saved as part of your Stella file.)

Part III: Sensitivity Analysis (Testing Multiple Values)

This feature will allow us to see more than one population graph at a time. We will alter the birth fraction five times to see how that affects the growth of the population.

39. While in Edit mode double click in the model window to open the Model Settings panel. Click on the "Sensitivity Analysis Setup" button at the bottom of the panel. 

40. Specify that you want 5 runs.

41. Add the variable "Birth Fraction" to the "Sensitivity Parameters" box using the same procedure as specified in step 23 for adding a variable to a graph.

⁴ Note: If you want to change the precision of the numbers displayed for a variable in the table, right click on the variable name in the table. Select "Format" at the bottom of the drop-down menu. Select the variable you want to format. For *Population* you may want to set "Precision" to 1 and "Scale by" to none.

42. Set “Distribution” to Ad-Hoc. Do you know what ad hoc means? Find out and give a definition here.


43. In the box next to “Ad-Hoc” values enter: 0.01, 0.02, 0.03, 0.045, 0.0535.

44. Use the *Birth Fraction* values to determine the number of children that are being born (per 1000 population) each year? (Fill out the table below.)

Birth Fraction (as a decimal)	Number of Newborns per 1000 population
0.01	
0.02	
0.03	
0.045	
0.0545	

45. Double click on the graph pad to open the graph panel. Make sure you are looking at the graph definition panel (designated by the wrench at the bottom of the panel) and not the graph styles panel (designated by the paint brush).

46. At the top of the panel click on the green + symbol to add another graph page to this graph pad. The top of the panel should now read “On page 2 of 2.” Add *Population* to the “Series List.” Set the “Min Value” for the graph to 0. Set the “Max Value” to 800000. Set the “Options” at the top of the panel to Comparative.⁵

47. To clear previous graphs press the “Restore All Devices” button  in the lower left part of the Stella window. Then press S-Run (same general location). Once the simulation is executed run five different graphs will appear, one for each different *Birth Fraction*.

48. After all the graphs are drawn, right click on each run name in the legend and change its name to identify the correct *Birth Fraction* value that produced the graph. (That is, change “Run 1” to “BF=.01,” “Run 2” to “BF=.02,” etc.) It is important to include this information on the graph pad so the reader knows what caused the graphs to be different.

49. To make sure the graphs are not overwritten by subsequent runs you will want to lock the graph. To do that right click on the graph pad. Select “Choose Visible Runs...” from the drop-down menu. Uncheck “Next Run.” Click Save. Print out the graph.⁶

⁵ Note: I prefer to have thicker lines on graphs. If you want to create graphs that contain thicker lines, specify the “Thickness” value as, say 2, instead of 1.

⁶ Whenever you are asked to print the model diagram or graph or table you will want to make sure all the information prints on one page. To show the page breaks that are set up in the software (in Edit mode) double click on the modeling widow to open the Model Settings panel. Under “Page Setting” click on Show page boundaries. Also make sure the “Page Rows” and “Page Columns” (at the bottom of the Model Settings panel) are set to the correct values so blank pages will not be printed.

50. What do you notice about the population growth shown by each of the five population graphs? (Relate the appearance of the graph to the specific number of newborns per 1000 population.)

51. For this question you must do research and *indicate where you found your answers*. Use numbers wherever possible to support your answers. Sources for information must be acceptable, so ask the teacher if you are unsure of the acceptability of the source. (Databases of world statistics and encyclopedias are acceptable sources.)

Name at least 2 countries that tend to have a low birth fraction. Give at least two good reasons (for each country) why the birth fraction would be low. Use characteristics about the countries you identified to support why the birth fraction is low for those specific countries.

Name at least 2 countries that tend to have a high birth fraction. Give at least two good reasons (for each country) why the birth fraction would be high. Use characteristics about the countries you identified to support why the birth fraction is high for those specific countries.

52. What sort of problems occur (name at least 2) when a country sustains a high birth fraction for several generations? (No research is required.)



Answer question J1 in your journal. Then pick either question J2 or J3 to also answer in your journal.

J1. What is the current United States birth fraction? Death fraction? Is the trend up or down? Is it different in different parts of the country? If it is different, why do you think there is a difference?

J2. What are the implications of China's policy on one birth per family. What effect might this have on future generations in China?

J3. Find an article or graph from the news dealing with population trends. Explain why the reporter thought the public should be concerned or interested in the trend. Paste the article in your journal. Make a note of the source and date for the article.